Photons evolution on very long distances

GOSSELIN P.

2018-07-20

Abstract

We lay down the fundamental hypothesis that any electromagnetic radiation transforms progressively, evolving towards and finally reaching after an appropriate distance the value of the cosmic microwave background radiation wavelength at $1,873$ *mm* or the frequency of $160,2$ GHz. This way we explain the cosmic redshift $\mathbb{Z}$ of far away Galaxies using only Maxwell's equations and the energy quantum principle for photons. This hypothesis is also true for wavelength longer or for frequency less than that of the cosmic microwave bacground. Hubble's law sprouts out naturally as the consequence of this transformation. According to this hypothesis we compute the Hubble constant using Pioneer satellite data and doing so deciphering the enigma of its anomalous behaviour. We speculate about a numerical composition of the Hubble constant and introduce the Hubble surface. This hypothesis helps to solve some cases that are still enigmatic for the standard cosmology. We show that our model fits exactly the distance modulus of cosmological candles and discuss the maximal observation distance of cosmological phenomena. We give an answer to the anomalous acceleration of the Pioneer satellite and we show that it is a universal constant common to any satellite.

Contents

List of Tables

List of Figures

1 Introduction

I nterpreting the redshift of the radiation coming from distant galaxies as a Doppler effect implies that those galaxies are moving away from the observer. In 1929 Edwin Hubble [11] [30] showed that the receding speed v of the observed galaxies was proportional to their distance d from the observer, was isotropic and related by the proportionality constant H_o. Recently this constant has been measured as 73 kilometers per second per Mega parsec using the most recent data collected with Cepheids, Cardona [29], with type I supernova, Dhawan [23] and including both with mega maser sources, Riess [22]. Hubble law is written as

$$v = dH_o \tag{1.1}$$

where v is the source receding speed, d its distance from the observer and H_o the proportionality constant. The redshift $\mathbb{Z}$ is a measure of this speed relative to light's vacuum speed c

$$\mathbb{Z} = \frac{v}{c} \tag{1.2}$$

so that the distance is given as

$$d = \frac{c\mathbb{Z}}{H_o} \tag{1.3}$$

Relative to the source wavelength λ_o and the observed wavelength λ, the redshift is

$$\mathbb{Z} = \frac{\lambda - \lambda_o}{\lambda_o} \tag{1.4}$$

Such interpretation infers an expanding universe since all observable objects seems to speed away the farther they are from the observer. Conversely this implies that $1/H_o$ years ago or approximately $13,7$ billions years, NASA [17], all the universe was embedded in a singularity that exploded to produce the expanding universe that we are observing today.

T he Doppler effect being the ratio of source's speed to light's speed and the fact that nothing can exceed the speed of light this ratio must always be lower than one. But it is common to observe galaxies showing $\mathbb{Z}$ ratios greater than one up to values of 12 according to the most recent observations, Bouwens [4], Brammer [5]. This goes against interpreting the redshift as Doppler caused and also against an expanding universe.

B ut since Hubble discovery [11] and coupled to Lemaître thesis [13] [14] [15] [24], nearly all cosmology theoreticians agree on a new kind of universe expansion. This is no more an explosion of matter into space but the more esoteric concept of a space expansion which is seen through the general relativity glasses. The expanding space idea explains the redshift by the stretching the light rays suffers during their travel through space. Considering the elasticity of space is mere speculation because there aren't any experiences possible to prove it. This is an open door to all kinds of exotic universe models and even to questioning the known observed properties of matter : Cameron [6], Terazawa [25]. Why only photons or electromagnetic waves should be dependent on the elastic geometry of the space while intrinsic dimensions of atoms, molecules and other material structures would not be?

O bserving light or photons in our local universe as well as in the laboratory shows us that photons are particles or waves that keep their properties indefinitely. On the contrary, atomic particles have a measurable lifetime and can decay into other particles. Since light speed is the maximum speed of any interaction that may happen in the universe this implies that photons cannot suffer any other action except to move. Then time doesn't exist for the photon and it is immutable.

N othing may suggest that physic's laws are different at large distance from us than in our local environment. If we agree on the fact that there aren't any difference between the local universe and the most remote one and also between, then it is advisable to consider that the photons might suffer a kind of transformation between the emission point and the observer. This way the redshift can be justified differently than by the stretching of space. The sole laboratory that can permit this verification is the universe itself since the billion of years required. We propose to explain the redshift and Hubble law through such a slow en route transformation of the electromagnetic radiation or photons. This is an intrinsic property not needing any intermediate mean or catalysis or interaction with matter. Then it must be conceded that the maximum speed of any interaction in the universe is a little bit higher than the photon speed. This limit might be very close to the current light speed because of the extremely long time required for photon transformation and the fact that all experiments done up to now are very well explained using the speed of light as the maximum speed limit of interactions in the universe. Then the photon is subject to structural transformation like any other denizen of the universe. This proposal seems to us much more acceptable and less esoteric than the elasticity of the space.

O n an other side, consider the electromagnetic radiation that comes from all parts of the sky. It has a wider spectrum much larger than the optical one. Particularly, radio astronomers A. Penzias and R. Wilson [21] in 1964 discovered a uniform and isotropic radiation at a microwave frequency of $160,2\ GHz$ or $1,873\ mm$ in wavelength corresponding to a temperature of $2,72548\ ^{\circ}K$, Fixen [10]. This radiation couldn't be associated with any object of the sky and it's presence has been explained as a residual of the Big Bang that happened 13,7 billions years ago. Very energetic photons emitted at that time might have lost their energy through space expansion ensuing the stretching of their wavelength. Today we would observe them as a cosmic microwave background residual (CMB).

W e already have proposed the transformation of the photons en route but this process mustn't proceed indefinitely, it must end somewhere. If it didn't, it would end up with an infinite number of photons of zero energy, an unacceptable situation in nature. We then propose that this endpoint of transformation happens when the photon energy reaches the CMB level. This way all radiation coming from everywhere melts in a kind of uniform fog that makes the physical limit of the observable world.

S ince the emission frequencies of photons are not limited to values higher than that of the CMB radiation, then photons emitted at a lower frequency shall transform too toward that CMB terminal limit. They shall evolve following an inverse direction that is towards higher frequencies. This phenomenon has not yet been observed since the absence of necessary radio astronomy infrastructure, more sophisticated radio telescope are required.

2 Theory

L et us consider a source of radiation emitted at a frequency higher than that of the CMB radiation. The global energy of the emitted flux decreases as a function of distance or time. Firstly, by the ejection of CMB photons (1,873 mm or 190,2 GHz) which disappear from the flux by melding with the local CMB radiation. Secondly, by a balancing of the number of photons which continuously increases while their common energy level decreases. The inverse situation happens when the radiation is of a frequency lower than that of the CMB radiation. A CMB photon is captured and added to the group that balances its energy by increasing its common frequency and decreasing its photon number. We proceed using Maxwell electromagnetic field equations and the quantum energy of the photon.

2.1 Extreme propagation

T he vacuum properties of an electromagnetic wave at very far distances are unknown to us. We suppose they are the same as they are locally meaning that Maxwell laws of electromagnetism are the same everywhere in the universe. Then for a plane wave of frequency ν, phase θ propagating at speed c in the direction $\vec{k}$, the electrical field $\vec{E}$ and the magnetic field $\vec{H}$ are dependent on distance d and time t.

$$\vec{E} = \vec{i}\, E_x(d,t) \tag{2.1}$$

$$E_x = E\ exp\ [j\omega(t - \frac{d}{c}) + \theta] \tag{2.2}$$

$$\vec{H} = \vec{j}\, H_y(d,t) \tag{2.3}$$

$$H_y = H\ exp\ [j\omega(t - \frac{d}{c}) + \theta] \tag{2.4}$$

The Poynting vector represents the energy flux carried by the wave

$$\vec{\mathbb{S}} = \vec{E}\ x\ \vec{H} \tag{2.5}$$

which is for the plane wave

$$\vec{\mathbb{S}} = \vec{k}\ \frac{E^2}{\mu_o c} \tag{2.6}$$

Between extremely distant points the Poynting vector cannot represent the energy conservation principle. A redshift is observed meaning a variation of the wavelength, an absent parameter of $\mathbb{S}$. Meanwhile the Poynting vector certainly represents the mean energy carried by the photons of the wave each being of energy

$$E = h\,\nu \tag{2.7}$$

where h is the Planck constant and ν is the photon frequency. When considering extremely long distances it might be appropriate to consider the variation of the photon density N and it's energy level so that the energy regime is preserved and the quantity

$$\xi = Nh\nu \tag{2.8}$$

is considered more representative on long distances where N and ν vary according to the distance d. Following our hypothesis, we consider a decrease of the energy density ξ as a function of distance. This decrease is proportional to the photon density. The proportionality constant k $[m^{-1}]$ is the energetic dissipation rate normalized with the CMB photon energy $h\nu_{cmb}$. We write

$$\frac{\partial \xi}{\partial d} = -\, k\, N_d\, h\, \nu_{cmb} \tag{2.9}$$

which gives

$$h\, (N_d \frac{\partial \nu_d}{\partial d} + \nu_d \frac{\partial N_d}{\partial d}) = -\, k\, N_d\, h\, \nu_{cmb} \tag{2.10}$$

$$N_d (k\, \nu_{cmb} + \frac{\partial \nu_d}{\partial d}) + \nu_d \frac{\partial N_d}{\partial d} = 0 \tag{2.11}$$

The solution to this differential equation is

$$N_d = \alpha\, e^{\frac{d}{\eta}} + C_1 \tag{2.12}$$

$$\nu_d = \beta\, e^{-\frac{d}{\eta}} + C_2 \tag{2.13}$$

where

$$\frac{\partial N_d}{\partial d} = \frac{\alpha}{\eta}\, e^{\frac{d}{\eta}} \tag{2.14}$$

$$\frac{\partial \nu_d}{\partial d} = -\frac{\beta}{\eta}\, e^{-\frac{d}{\eta}} \tag{2.15}$$

giving

$$\alpha k \nu_{cmb} e^{\frac{d}{\eta}} + k \nu_{cmb} C_1 - \frac{\beta C_1 e^{-\frac{d}{\eta}}}{\eta} + \frac{C_2 \alpha e^{\frac{d}{\eta}}}{\eta} = 0 \tag{2.16}$$

This last equation being true for any value of d implies

$$C_1 = 0 \tag{2.17}$$

$$C_2 = -\eta k \nu_{cmb} \tag{2.18}$$

The limiting conditions are at $d = 0 : N_d = N_o$, $\nu_d = \nu_o$ so that

$$\alpha = N_o \tag{2.19}$$

$$\beta = \nu_o + \eta k \nu_{cmb} \tag{2.20}$$

and finally

$$N_d = N_o\, e^{\frac{d}{\eta}} \tag{2.21}$$

$$\nu_d = (\nu_o + \eta k \nu_{cmb})\, e^{-\frac{d}{\eta}} - \eta k \nu_{cmb} \tag{2.22}$$

$$\nu_d = \nu_o e^{-\frac{d}{\eta}} + \eta k \nu_{cmb} (e^{-\frac{d}{\eta}} - 1) \tag{2.23}$$

where the wavelength is

$$\lambda_d = \frac{\lambda_o}{\frac{\eta k \lambda_o}{\lambda_{cmb}}(e^{-\frac{d}{\eta}} - 1) + e^{-\frac{d}{\eta}}} \tag{2.24}$$

Introducing the redshift at distance d

$$\mathbb{Z}_d = \frac{\nu_o - \nu_d}{\nu_d} \tag{2.25}$$

and when the radiation has been completely transformed into CMB radiation

$$\mathbb{Z}_{cmb} = \frac{\nu_o - \nu_{cmb}}{\nu_{cmb}} \tag{2.26}$$

The wavelength is then written as

$$\lambda_d = \frac{\lambda_o}{\frac{\eta k}{\mathbb{Z}_{cmb}+1}(e^{-\frac{d}{\eta}} - 1) + e^{-\frac{d}{\eta}}} \tag{2.27}$$

the frequency

$$\nu_d = \nu_o \left[(1 + \frac{\eta k}{\mathbb{Z}_{cmb}+1}) e^{-\frac{d}{\eta}} - \frac{\eta k}{\mathbb{Z}_{cmb}+1} \right] \tag{2.28}$$

and the distance using (2.22)

$$d = \eta \; ln \frac{\nu_o + \eta k \nu_{cmb}}{\nu_d + \eta k \nu_{cmb}} \tag{2.29}$$

$$d = \eta \; ln \frac{(\mathbb{Z}_{cmb}+1) + \eta k}{\frac{(\mathbb{Z}_{cmb}+1)}{(\mathbb{Z}_d+1)} + \eta k} \tag{2.30}$$

When the source's frequency is lower than the CMB one, the sign of k (2.9) and η (2.12), (2.13) must be changed while the sign of the product ηk does not.

Since the transformation of the photons happens on very long distances, it seems normal that η, the transformation constant, be numerically very large, so for short distances we may consider the serial development of the exponential, keeping only the first two terms of the series. Using equation (2.22), we get

$$e^{\frac{d}{\eta}} = (\nu_o + \eta k \nu_{cmb})/(\nu_d + \eta k \nu_{cmb}) \approx 1 + d/\eta \tag{2.31}$$

$$d = \eta \; (\nu_o - \nu_d) \; / \; (\nu_d + \eta k \nu_{cmb}) \tag{2.32}$$

$$d = \eta \; (\nu_o - \nu_d) \; / \; [\nu_d (1 + \frac{\eta k \nu_{cmb}}{\nu_d})] \tag{2.33}$$

$$d = \frac{\eta \mathbb{Z}_d}{1 + \eta k \frac{\mathbb{Z}_d + 1}{\mathbb{Z}_{cmb}+1}} \tag{2.34}$$

Considering the local universe, that is on very short distances, the energy density hasn't time to change and is considered constant (B.8) so that its rate of decrease is null

(2.9). Therefore the energy dissipation parameter k is zero and the distance (2.34) is written

$$d = \eta \mathbb{Z}_d \tag{2.35}$$

Here we have Hubble's law and to have it's classical expression, we write

$$\eta = c/H_o \tag{2.36}$$

Replacing k and η in (2.30), in the local environment, the distance is

$$\boxed{d = \frac{c}{H_o} ln\,(\mathbb{Z}_d + 1)} \tag{2.37}$$

the wavelength (2.27) is

$$\lambda_d = \lambda_o e^{\frac{dH_o}{c}} = \lambda_o(\mathbb{Z}_d + 1) \tag{2.38}$$

and the photon density (2.21) is

$$N_d = N_o e^{\frac{dH_o}{c}} = N_o(\mathbb{Z}_d + 1) \tag{2.39}$$

N.B. You may find interesting, in appendix, two variations on the method used to obtain these results in the local environment.

2.2 Frequency variation

L et us examine how the frequency (2.22) change under a variation of distance or travel time $d = ct$. Taking the time derivative

$$\partial\nu_d/\partial t = (-c/\eta)(\nu_d + \eta k \nu_{cmb}) \tag{2.40}$$

$$\partial\nu_d/\partial t = -c\nu_d/\eta - kc\nu_{cmb} \tag{2.41}$$

Replacing η by its value c/H_o

$$\partial\nu_d/\partial t + kc\nu_{cmb} = -\nu_d H_o \tag{2.42}$$

we get the Hubble constant as

$$\boxed{H_o = -\frac{\partial\nu_d/\partial t}{\nu_d} - \frac{kc\nu_{cmb}}{\nu_d}} \tag{2.43}$$

or

$$H_o = -\frac{\partial\nu_d/\partial t}{\nu_d} - kc\frac{(\mathbb{Z}_d + 1)}{(\mathbb{Z}_{cmb} + 1)} \tag{2.44}$$

2.3 The flux

N ow, let us consider a divergence free flux of photons moving on a very long distance where energy is conserved. Consider an initial volume V_o enclosing a photon density N_o all of the same wavelength λ_o and similarly at a far away distance d, the same elements, a volume V_d enclosing a photon density N_d all of the same wavelength λ_d. Using equations (2.38), (2.39), the total energy in each of those volumes is

$$E_o = N_o V_o h\nu_o = N_o V_o hc/\lambda_o \; [joule] \tag{2.45}$$

$$E_d = N_d V_d h\nu_d = N_o(\mathbb{Z}+1)V_d hc/[\lambda_o(\mathbb{Z}+1)] = N_o V_d hc/\lambda_o \; [joule] \tag{2.46}$$

We can then say that equal volumes have equal energy quantity. The energy density is the same in both cases.

$$\rho_o = E_o/V_o = E_d/V_d = \rho_d \; [joule/meter^3] \tag{2.47}$$

The energy density is conserved while the photon density increases.

L et us consider a photon flux S crossing normally a surface s. After a time t, n of those photons will occupy a volume

$$V = sct \; [meter^3] \tag{2.48}$$

where the photon density is

$$N = \frac{n}{sct} \; [photons/meter^3] \tag{2.49}$$

Each photon being of energy $h\nu$, the total energy E in the volume V is

$$E = nh\nu = Nscth\nu \; [joule] \tag{2.50}$$

This energy having crossed the surface s during time t, the flux is

$$S = \frac{E}{st} = Nch\nu = \frac{Nhc^2}{\lambda} \; [joule/second/meter^2] \; or \; [watt/meter^2] \tag{2.51}$$

The specific flux f_ν is defined as the ratio of the flux per frequency unit

$$f_\nu = \frac{S}{\nu} = Nch \; [joule/meter^2] \tag{2.52}$$

whose unit of measure is the Jansky

$$1 \; Jansky = 10^{-26} \; [joule/meter^2] \; or \; [watt/meter^2/hertz] \tag{2.53}$$

The corresponding photon density is

$$N = \frac{f_\nu}{ch} \; [photons/meter^3] \tag{2.54}$$

and the energetic density is

$$\rho = Nh\nu = \frac{\nu f_\nu}{c} = \frac{S}{c}\ [joule/meter^3] \tag{2.55}$$

The power density is defined as f_λ as the ratio of the flux per unit of wavelength

$$f_\lambda = \frac{S}{\lambda} = \frac{\nu f_\nu}{\lambda} = \frac{\nu^2 f_\nu}{c} = \frac{c f_\nu}{\lambda^2} = \frac{c\rho}{\lambda} = \nu\rho\ [watt/meter^3] \tag{2.56}$$

$$f_\nu = \frac{\lambda^2 f_\lambda}{c} = \frac{c f_\lambda}{\nu^2} \tag{2.57}$$

$$S = \lambda\, f_\lambda = \nu\, f_\nu \tag{2.58}$$

T he electromagnetic wave transforms itself on long distances and consequently the number and the frequency of the photons does too so let us look at the flux between an origin o and distance d. The specific flux at origin is according to (B.33))

$$^o f_\nu = {^o N_o} ch \tag{2.59}$$

and at distance d

$$^d f_\nu = {^d N_o} ch = {^o N_o}(\mathbb{Z}+1)ch = {^o f_\nu}(\mathbb{Z}+1) \tag{2.60}$$

The flux S (2.58) at origin is

$$^o S = \nu_o {^o f_\nu} \tag{2.61}$$

and at distance d

$$^d S = \nu_d {^d f_\nu} = [\nu_o/(\mathbb{Z}+1)][{^o f_\nu}(\mathbb{Z}+1)] = \nu_o {^o f_\nu} \tag{2.62}$$

We see that the flux is conserved on distance while the specific flux adjust itself. The same way the energy density doesn't change under distance while the power density adjust.

$$\boxed{{^o S} = {^d S} = \nu_o {^o f_\nu} = \nu_d {^d f_\nu} = \lambda_o {^o f_\lambda} = \lambda_d {^d f_\lambda} = {^o \rho} c = {^d \rho} c} \tag{2.63}$$

2.4 Beyond the cosmic microwave background (CMB)

A ccording to our hypothesis, photons whose frequency is higher than that of the CMB radiation loose their energy as they travel and their frequency decrease or equivalently their wavelength increase until they reach the CMB level. In that case, a shift of the wavelength is observed towards the red side of the spectrum. Conversely, photons whose frequency is lower than that of the CMB radiation gain energy as they travel and their frequency increase or equivalently their wavelength decrease until they reach the CMB level. In that case, a shift of the wavelength is observed towards the blue side of the spectrum. In order to make a clear distinction between a cosmological effect from a Doppler effect caused by the intrinsic movement of stellar objects, we use the term "cosmological shift" instead of "red shift". The cosmological shift shall be > 0 or < 0 while the redshift will be towards the red or blue side of the spectrum. For wavelengths shorter than the CMB wavelength, the cosmic shift is a positive value greater than zero $\mathbb{Z}_d > 0$ and up to the maximum value infinity for zero length wavelengths $\lambda_o = 0$. For wavelengths longer than the CMB wavelength, the cosmic shift is a negative value $\mathbb{Z}_d < 0$ which may reach the minimal value of -1 for infinite wavelengths $\lambda_o = \infty$.

2.5 Link between Visual and Radiometric radiation

L et a source send radiation at a wavelength shorter than the CMB a visual one, $\lambda_{o,v}$. Let also that source send radiation at a wavelength longer than the CMB a radiometric one, $\lambda_{o,r}$. Those two radiations, having travelled a distance d, transform according to equation (2.38) :

$$\lambda_{d,v} = \lambda_{o,v}\; e^{\frac{dH_o}{c}} \tag{2.64}$$

$$\lambda_{d,r} = \lambda_{o,r}\; e^{-\frac{dH_o}{c}} \tag{2.65}$$

and since they have a common distance, we can eliminate that term in those equations to get :

$$\lambda_{o,v}\,\lambda_{o,r} = \lambda_{d,v}\,\lambda_{d,r} = Constant \tag{2.66}$$

Using the visual $\mathbb{Z}_v$ and radiometric $\mathbb{Z}_r$ cosmic shift, this equation can be rewritten as :

$$\boxed{\mathbb{Z}_r = \frac{-1}{1+\frac{1}{\mathbb{Z}_v}}} \tag{2.67}$$

This expression is symmetric and the $\mathbb{Z}$ can be interchanged.

We must take care not to confound this radio cosmic shift $\mathbb{Z}_r$ with the radio redshift used by radio astronomers Z_{Radio} who use a different definition for the recession speed and for the redshift. They define the source radio speed V_{Radio} as:

$$V_{Radio} = c\,Z_{Radio} = c(\nu_o - \nu)/\nu_o \tag{2.68}$$

while the classic definition for optical recession speed is:

$$V_{Optic} = c\,Z_V = c(\nu_o - \nu)/\nu \tag{2.69}$$

The relation between those two is:

$$Z_{Radio} = 1/(1+(1/Z_V)) \tag{2.70}$$

We note the following link between those two different radio shifts, (2.67) and (2.70), issued from two different concepts:

$$Z_{Radio} = -\mathbb{Z}_r \tag{2.71}$$

3 The Hubble constant

S ince its discovery, the Hubble constant has been subject to much controversy about its most probable value, a low or a high value, the span being from 50 $km\ s^{-1}\ Mpc^{-1}$ to 100 $km\ s^{-1}\ Mpc^{-1}$. The true value relies essentially upon the precise measurement of distances which are always obtained through indirect methods. The observation of Cepheids stars is still the most appropriate mean for this measurement. It is based on the Leavitt's law where Cepheid stars shows a periodic luminosity related to their maximum luminosity, giving an indirect measure of their distance. For this reason those stars constitute standard candles which constitute a local distance scale. To go beyond the local universe this scale must be extended beyond the limit of our galaxy. Actually we use the properties of supernova type Ia (SNIa), water maser sources and binary eclipsing stars. Those objects extend the distance scale and its precision improves with new measurement techniques. The most recent evaluations by Riess [22] indicate a value of $H_o = 73,02\ \pm 1,79\ km\ s^{-1}\ Mpc^{-1}$ or $2,366417\ x\ 10^{-18}\ s^{-1}$

3.1 Pioneer

T he Pioneer 10 satellite has been decelerating constantly since it's departure from the solar system and still was when communications ended due to the loss of strength of the signal. The Doppler signal measuring the satellite speed drifted constantly showing a deceleration of the satellite. Since the satellite was out of solar bounds it should have kept a constant speed and up to now no satisfactory explanation has been given to this phenomena. Turyshev and Toth [27]

T he satellite distance and speed were measured very precisely through the use of a S band signal of frequency $\sim 2,1\ GHz$ sent from earth station and returned as $\sim 2,3\ GHz$ by the satellite in such a way that the stability and precision of the signal were independent of the satellite equipment. The satellite being out of solar bounds should have moved ballistically according to the classic mechanical laws. Throughout the whole journey, a constant frequency drift of $5,99\ \pm\ 0,01\ x\ 10^{-9}\ Hz\ s^{-1}$ has been observed toward a higher one. Interpreted as a Doppler shift, it is equivalent to a satellite deceleration of $8,74\ \pm\ 1,33\ x\ 10^{-10}\ m\ s^{-2}$. We consider that this variation is nothing else than the effect of the transformation of the electromagnetic signal according to our model.

C learly if the satellite slows down, one will observe a blue shift of the Doppler signal which is already red shifted because of the satellite receding speed. Newtonian mechanic tell us that the satellite doesn't slow down but moves at constant speed. The signal round trip is increasing at a constant pace and according to our model, the signal must suffer a constant change. Since the mean frequency of the signal we estimate at $\sim 2,19\ GHz$ is lower than the cosmic microwave background of $160,2\ GHz$, the frequency of the signal must increase or equivalently the wavelength shorten. So the observed blue shift drift owing to the continuous increasing signal round trip distance. And the false impression of a slowing down of the satellite. The enigmatic behaviour of the Pioneer satellite is then explained and by the way confirms our hypothesis of the electromagnetic wave transformation on long distances.

3.2 k and H_o

T he Pioneer satellite signal drift enable us to evaluate the dissipation coefficient k. Taking into account the change of sign of k and η, in equation (2.41), we have

$$k = (\partial \nu_d/\partial t - \nu_d H_o)/(c\ \nu_{cmb}) \tag{3.1}$$

The value of the dissipation coefficient is

$$k = \frac{5,99\ x\ 10^{-9} - 2,19\ x\ 10^{9}\ x\ 2,366417\ x\ 10^{-18}}{2,997925\ x\ 10^{8}\ x\ 1,602\ x\ 10^{11}} \tag{3.2}$$

$$k = 1,681453\ x\ 10^{-29} m^{-1} \tag{3.3}$$

In the local universe, on very small scale, which is the case with Pioneer satellite, the dissipation coefficient may be considered as null $k = 0$ and equation (3.1) shows us the local Hubble constant as

$${}^{k0}H_o = \frac{\dot{\nu}_d}{\nu_d} \tag{3.4}$$

$${}^{k0}H_o = \frac{5,99\ x\ 10^{-9}}{2,19\ x\ 10^{9}} = 2,735159\ x\ 10^{-18}\ s^{-1} = 84,39\ km\ s^{-1}\ Mpc^{-1} \tag{3.5}$$

This value is very close to 85 $\pm$ 5 $km\ s^{-1}\ Mpc^{-1}$ found by Willick [3] from Cepheids measurements.

W e find interesting to evaluate the ratio of the Hubble constant measured by Riess [22] and the local value. This ratio is close to half the square root of three

$$H_o\ /\ {}^{k0}H_o = 2,366417\ x\ 10^{-18}/2,735159\ x\ 10^{-18} = 73,02/84,39 \tag{3.6}$$

$$H_o\ /\ {}^{k0}H_o = 0,865184 \sim \sqrt{3}/2 = \sin(\pi/3) \tag{3.7}$$

The polemic around the value of the Hubble constant seems to be only due to a difference between a local measure versus a long distance measure.

3.3 Breaking down

W hen feasible, it is of interest to consider a constant as a combination of fundamental ones. We express the Hubble constant as a function of fundamental constants doing so as to cope with the units of measure and searching for a value the closest as possible to the currently measured value. The found composition is

$${}^{k0}H_o = \frac{\alpha\ R_\infty^2\ \left(\frac{\hbar\ G}{c}\right)^{\frac{1}{2}}}{(2\pi)^4} \tag{3.8}$$

where α is the fine structure constant, R_∞ is the Rydberg constant, $\hbar$ is the reduced Planck constant, G is the universal gravitational constant and c is the vacuum speed of light. Using the values of the fundamental constants as given by CODATA [8], Wikipedia [31] also available in appendix A, we find for the local Hubble constant

$$^{k0}H_o = 2,731933\; x\; 10^{-18}\; s^{-1} = 84,2987\; km\; s^{-1}\; Mpc^{-1} \tag{3.9}$$

Considering equations (3.6) and (3.7), we define the Hubble constant as

$$\boxed{H_o = \sin(\pi/3)\; \frac{\alpha\, R_\infty^2\; \left(\frac{\hbar\, G}{c}\right)^{\frac{1}{2}}}{(2\pi)^4}} \tag{3.10}$$

$$H_o = 2,365923\; x\; 10^{-18}\; s^{-1} = 73,00\; km\; s^{-1}\; Mpc^{-1} \tag{3.11}$$

This value is the same as published by Riess [22] : $73,02 \pm 1,79\; km\; s^{-1}\; Mpc^{-1}$

S ince that frequencies used for the communications between the Pioneer satellite and earth aren't known precisely the dissipation coefficient may vary somehow (3.2), we then arbitrarily make its value as

$$\boxed{k = \frac{5}{3}\; x\; 10^{-29}\; m^{-1}} \tag{3.12}$$

and the Hubble length too is defined as

$$\boxed{\eta = 1,267127\; x\; 10^{26}\; m = 4,1065\; Gpc} \tag{3.13}$$

Some useful products are

$$k\, c = 5\; x\; 10^{-21}\; s^{-1} \tag{3.14}$$

$$\eta\, k = 2,112\; x\; 10^{-3} \tag{3.15}$$

$$\eta\, k\, \nu_{cmb} = 338,3\; MHz \tag{3.16}$$

3.4 The Hubble length and surface

I ntroducing the Planck length

$$\ell_p = \left(\frac{G\hbar}{c^3}\right)^{\frac{1}{2}} \tag{3.17}$$

into the previous equation (3.8), the local Hubble length is

$$^{k0}\eta = \frac{c}{^{k0}H_o} = \frac{(2\pi)^4}{\alpha R_\infty^2 \ell_p} \tag{3.18}$$

and the Hubble length is

$$\eta = \frac{c}{H_o} = \frac{c}{\sin(\pi/3)\ {}^{k0}H_o} = \frac{{}^{k0}\eta}{\sin(\pi/3)} \tag{3.19}$$

We define the following reduced constants

$$\tilde{\alpha} = \frac{\alpha}{2\pi} \tag{3.20}$$

$$\tilde{R}_\infty = \frac{R_\infty}{2\pi} \tag{3.21}$$

$$\tilde{\ell}_p = \frac{\ell_p}{2\pi} \tag{3.22}$$

giving us a nicer writing of the local Hubble length

$${}^{k0}\eta = \left(\tilde{\alpha}\, \tilde{R}_\infty^2\, \tilde{\ell}_p\right)^{-1} \tag{3.23}$$

The Hubble length is $1,267127\ x\ 10^{26}$ meters or $4,1065\ Gpc$ and the local value is $1,097364\ x\ 10^{26}$ meters or 3,5563 Gpc.

W e observe that the digits of the local Hubble length are nearly the same as those of the Rydberg constant $1,0973731568539(55)\ x\ 10^7\ m^{-1}$. We then define the reduced local Hubble surface ${}^{k0}\tilde{\sigma}_H$ as the ratio of local Hubble length to Rydberg constant

$${}^{k0}\tilde{\sigma}_H = \frac{{}^{k0}\eta}{R_\infty} \tag{3.24}$$

$${}^{k0}\tilde{\sigma}_H = 10^{19}\ m^2 \tag{3.25}$$

The corresponding local Hubble surface ${}^{k0}\sigma_H$ is

$${}^{k0}\sigma_H = 2\pi\ {}^{k0}\tilde{\sigma}_H \tag{3.26}$$

$$\boxed{{}^{k0}\sigma_H = \left(\tilde{\alpha}\, \tilde{R}_\infty^3\, \tilde{\ell}_p\right)^{-1} = 2\pi\ 10^{19}\ m^2} \tag{3.27}$$

The local Hubble surface may correspond to simple geometric surfaces. For example it is a sphere whose radius is $\sqrt{5}$ Giga meters or 2 236 068 kilo meters or $0,015$ AU that is $3,21$ solar radii.

Units of measure	Symbol	Value	Square (side)	Disc (radius)	Sphere (radius)
Meter	m	1	$7,93\ x\ 10^9$	$4,47\ x\ 10^9$	$2,24\ x\ 10^9$
Earth-Moon	EA	$3,84\ x\ 10^8$	$20,62$	$11,63$	$5,82$
Sun radius	SR	$6,96\ x\ 10^8$	$11,39$	$6,43$	$3,21$
Astronomical unit	UA	$1,496\ x\ 10^{11}$	$0,053$	$0,03$	$0,015$

Table 1: Hubble equivalent surfaces

4 Solved enigmas

M ore and more deviations or unexplained effects pop up in the context of an expansionist cosmology. Some of those phenomenons are very well explained by our model.

4.1 Receding speed of the Cepheids

U sing the redshift definition (1.2) with the distance as given by equation (2.37)

$$d = \frac{c}{H_o}\ ln(\frac{v}{c} + 1) \tag{4.1}$$

recession speed is

$$v = c\ (e^{\frac{dH_o}{c}} - 1) \tag{4.2}$$

The apparent recession speed is exponential and not linear (1.1). If a linear relation is used when observing objects situated at farther and farther distances or increasing red-shifts, higher and higher values of the Hubble constant H_o will be found. This explains the measured differences between Cepheid close to us and others farther from us. This fact is shown and discussed in the paper of Arp [2] where he looks for an explanation by an excess of redshift for distant Cepheids. Next figure reproduces figure 4 of Arp's paper where the increasing values of the Hubble constant as a function of distance are clearly seen.

4.2 Cosmic microwave background and supernova

Y ershov [20] showed the presence of a high correlation between the local increase of the cosmic microwave background temperature T_{sn} at supernova positions and the redshift of those supernova $\mathbb{Z}_{sn}$. Looking at SN type Ia he finds that the temperature increases as $T_{sn} = 58,0 \pm 9,0\ \mathbb{Z}_{sn}\ [\mu K]$. This local temperature excess is proportional to the associated redshift of those supernova. The expansionist cosmology cannot explain this phenomenon. However this effect confirms our transformation model of

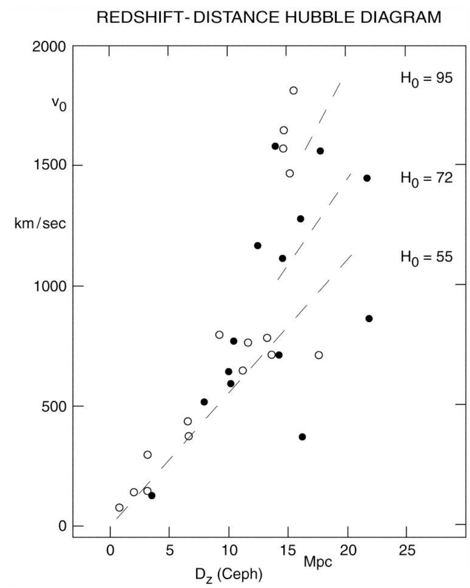

Figure 1: Various values of the Hubble constant

the electromagnetic energy as a function of distance. At those supernova spots, there is always an excess of temperature over the cosmic background. And this increase is directly proportional to the source's distance or its cosmic shift. This temperature increase is proportional to the source distance since the farther it is a higher fraction of the energy spectrum is transformed to the CMB level. In fact photons of any wavelength can't transform farther than their proper CMB distance d_{cmb}. Then all the spectrum energy which has a d_{cmb} less than the distance to this emitter is converted into CMB radiation. This create an accumulation of energy at this spot. It follows that at such observation point a local excess of the CMB is observed and this excess is proportional to the distance of this point as measured by its cosmic-shift.

5 Distance

T he electromagnetic wave transformation along distance implies a new distance modulus formula and new maximum distances of the observable world.

5.1 Distance modulus

L et consider a point source of power L_o which emits radially a photon flux. A spherical thin sheet of infinitesimal thickness ε centered on that source and at distance f has a photon density N_f. Similarly an infinitesimally thin sheet at distance d has a photon density N_d. Since the photons are conserved, their number in both sheets is the same so we write

$$4\,\pi\,d^2\,\varepsilon\,N_d = 4\,\pi\,f^2\,\varepsilon\,N_f \tag{5.1}$$

$$N_d = N_f\,\frac{f^2}{d^2} \tag{5.2}$$

But on very long distances, photons transforms themselves between distances f and d as 2.21 and 2.23, so

$$N_d = N_f\,e^{\frac{(d-f)}{\eta}} \tag{5.3}$$

$$\nu_d = \nu_f\,e^{-\frac{(d-f)}{\eta}} - \eta\,k\,\nu_{cmb}\,(1-e^{-\frac{(d-f)}{\eta}}) \tag{5.4}$$

Then the photon density at d is

$$N_d = N_f\,e^{\frac{(d-f)}{\eta}}\,\frac{f^2}{d^2} \tag{5.5}$$

And taking int account the frequency change, the product $N_d\nu_d$ is

$$N_d\,\nu_d = N_f\,[\nu_f\,e^{-\frac{(d-f)}{\eta}} - \eta\,k\,\nu_{cmb}\,(1-e^{-\frac{(d-f)}{\eta}})]\,e^{\frac{(d-f)}{\eta}}\,\frac{f^2}{d^2} \tag{5.6}$$

The energy flux at those surfaces is

$$S_f = N_f\,\nu_f\,h \tag{5.7}$$

$$S_d = N_d\,\nu_d\,h = N_f\,[\nu_f\,e^{-\frac{(d-f)}{\eta}} - \eta\,k\,\nu_{cmb}\,(1-e^{-\frac{(d-f)}{\eta}})]\,e^{\frac{(d-f)}{\eta}}\,\frac{f^2}{d^2}\,h \tag{5.8}$$

and their ratio is

$$\frac{S_d}{S_f} = \frac{f^2}{d^2}\cdot[1 + \frac{\eta\,k\,\nu_{cmb}}{\nu_f}\,(1-e^{\frac{(d-f)}{\eta}})] \tag{5.9}$$

According to the definition of the apparent magnitude, the difference taken at those distances is

$$m_d - m_f = -\,2,5\,log\left\{\frac{S_d}{S_f}\right\} \tag{5.10}$$

$$m_d - m_f = -\,2,5\,log\left\{\frac{f^2}{d^2}\cdot[1 + \frac{\eta\,k\,\nu_{cmb}}{\nu_f}\,(1-e^{\frac{(d-f)}{\eta}})]\right\} \tag{5.11}$$

At the reference distance $f = 10\ pc$, m_f is defined as the absolute magnitude M and this difference defines the distance module μ of the source so

$$\mu = m - M \tag{5.12}$$

$$\mu_d = m_d - m_{10pc} \tag{5.13}$$

$$\mu_d = -5\ log\ f\ + 5\ log\ d\ - 2,5\ log\ \{1\ +\ \frac{\eta\ k\ \nu_{cmb}}{\nu_f}\ (1 - e^{\frac{(d-f)}{\eta}})\} \tag{5.14}$$

$$\mu_d = -5\ + 5\ log\ d\ - 2,5\ log\ \{1\ +\ \frac{\eta\ k\ \nu_{cmb}}{\nu_{10_{pc}}}\ (1 - e^{\frac{(d-10pc)}{\eta}})\} \tag{5.15}$$

As it will be shown in a following section η is the Hubble length that is the ratio $\frac{c}{H_o}$ whose value is Giga parsec size. This makes exponential $\frac{f[pc]}{\eta[Gpc]}$ unitary. Introducing the cosmic redshift (2.25) and (2.26), we write

$$\mu_d = -5\ + 5\ log\ d\ - 2,5\ log\ \{1\ +\ \frac{\eta\ k}{(\mathbb{Z}_{cmb}+1)}\ (1 - e^{\frac{d}{\eta}})\} \tag{5.16}$$

where we used the value $\mathbb{Z}_{10_{pc}} = 0$. Helped with the distance equation (2.30), the distance modulus becomes

$$\begin{aligned} \mu_d = & -5 \\ & +5\ log\left\{\eta\ ln\left\{\frac{(\mathbb{Z}_{cmb}+1)+\eta k}{\{\frac{(\mathbb{Z}_{cmb}+1)}{(\mathbb{Z}_d+1)}+\eta k\}}\right\}\right\} \\ & -2,5\ log\ \{\frac{(\mathbb{Z}_{cmb}+1)+\eta\ k}{(\mathbb{Z}_{cmb}+1)+\eta\ k\ (\mathbb{Z}_d+1)}\} \end{aligned} \tag{5.17}$$

$$\begin{aligned} \mu_d = & -5 \\ & +5\ log\ \eta \\ & +5\ log\left\{ln\{(\mathbb{Z}_d+1)\} + ln\left\{\frac{(\mathbb{Z}_{cmb}+1)+\eta\ k}{(\mathbb{Z}_{cmb}+1)+\eta\ k\ (\mathbb{Z}_d+1)}\right\}\right\} \\ & -2,5\ log\left\{\frac{(\mathbb{Z}_{cmb}+1)+\eta\ k}{(\mathbb{Z}_{cmb}+1)+\eta\ k\ (\mathbb{Z}_d+1)}\right\} \end{aligned} \tag{5.18}$$

With the constants values, $H_o = 2,365923x10^{-18}\ s^{-1} = 73,00\ Km\ sec^{-1}\ Mpc^{-1}$ (3.11), $\eta = 1,267127x10^{26}\ m = 4,1065\ Gpc$ (3.12), $k = 5/3x10^{-29}\ m^{-1}$ (3.13), $\eta\ k\ =\ 2,112\ x\ 10^{-3}$ (3.15), the distance modulus reduces to

$$\begin{aligned} \mu_d = & -5,0 \\ & +48,067359 \\ & +5,0\ log\ \{ln\ [(\mathbb{Z}_d+1)] + \\ & \qquad ln\ [(\mathbb{Z}_{cmb}+1)+0,002112] - \\ & \qquad ln\ [(\mathbb{Z}_{cmb}+1)+0,002112\ (\mathbb{Z}_d+1)]\ \} \\ & -2,5\ log\ \{(\mathbb{Z}_{cmb}+1)+0,002112\} \\ & +2,5\ log\ \{(\mathbb{Z}_{cmb}+1)+0,002112\ (\mathbb{Z}_d+1)\} \end{aligned} \tag{5.19}$$

Keeping simple and neglecting small quantities which correspond to the local environment where $k = 0$, the distance modulus reduces to

$$\boxed{\mu_d = 43,067359 + 5 \; log \; ln \; (\mathbb{Z}_d + 1)} \tag{5.20}$$

When the source emits at wavelength longer than the cosmic microwave background λ_{cmb}, the cosmic redshift $\mathbb{Z}_d$ is a negative value greater than -1. This formula is a little different from the classical distance modulus of the Hubble law which is

$$\mu_d = 43,067359 + 5 \; log \; \mathbb{Z}_d \tag{5.21}$$

As a function of the redshift $\mathbb{Z}$, figure 2 shows distances d, top curves, and distance modulus, bottom curves, according to our model and Hubble law. Table 2 lists various values where we have included those for the expansion model from Nick Gnedin computing form (http://home.fnal.gov/~gnedin/cc/) with $H_o = 73,0$ and $\Omega_o = 0,315$.

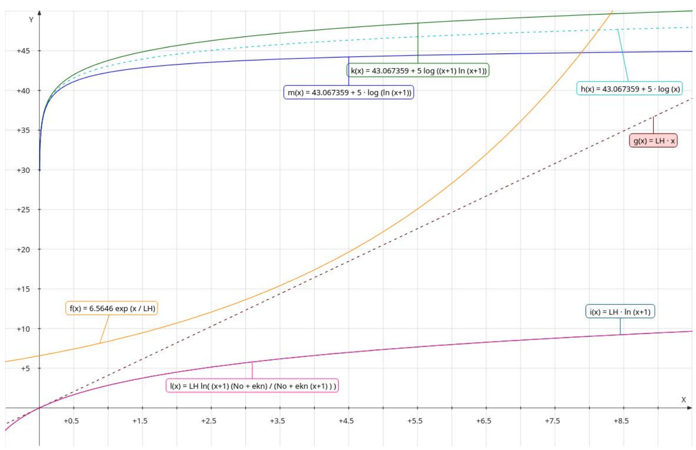

Figure 2: Figure 2. Distance modulus and distance vs cosmic shift.

5.2 Hubble Diagram

T he discovery of the Hubble law put forward many cosmological models trying to justify its nature. Being linear on short distances this law becomes exponential on very long distances. The development of new tools and methods brought a new distance scale calibrated upon Céphеids, Supernovae SNIa and gamma ray bursts. All the collected data makes the Hubble diagram whose general trend follows an exponential law. Analysis of this data by Marosi [16] shows that the best fitting function of the distance modulus μ, corrected for extinction, as a function of the redshift $\mathbb{Z}$ for 280 supernovae type SNIa covering the interval $\mathbb{Z} = 0,0141 - 8,1$ is the exponential

$$\mu = 44,109769 \; \mathbb{Z}^{\;0,059883} \tag{5.22}$$

Cosmic-shift	Distance	Distance modulus	Classical distance	Classical modulus	Distance expansion model	Module Nick Gnedin computations
	(2.30)	(5.21)	(1.3)	(5.22)		
$\mathbb{Z}_d$	d_{Gpc}	μ	d_{Gpc}	μ	d_{Gpc}	μ
-0,99999	47,28	48,37				
-0,9999	37,82	47,89				
-0,999	28,37	47,26				
-0,99	18,91	46,38				
-0,9	9,46	44,88				
-0,8	6,61	44,10				
-0,7	4,94	43,47				
-0,6	3,76	42,88				
-0,5	2,85	42,27				
-0,4	2,10	41,61				
-0,3	1,46	40,83				
-0,2	0,92	39,81				
-0,1	0,43	38,18				
0,1	0,39	37,96	0,41	38,07	0,44	38,24
0,2	0,75	39,37	0,82	39,57	0,94	39,87
0,3	1,08	40,16	1,23	40,45	1,49	40,87
0,4	1,38	40,70	1,64	41,08	2,09	41,60
0,5	1,67	41,11	2,05	41,56	2,27	42,17
1	2,85	42,27	4,11	43,07	6,32	44,01
2	4,51	43,27	8,21	44,57	14,87	45,86
3	5,69	43,78	12,32	45,45	24,30	43,96
4	6,61	44,10	16,43	46,08	34,25	47,67
5	7,36	44,33	20,53	46,56	44,56	48,24
6	7,99	44,51	24,64	46,96	55,13	48,71
7	8,54	44,66	28,75	47,29	65,92	49,10
8	9,02	44,78	32,85	47,58	76,88	49,43
9	9,46	44,88	36,96	47,84	87,96	49,72
10	9,85	44,97	41,07	48,07	99,16	49,98
20	12,50	45,48	82,13	49,57	215,35	51,67
30	14,10	45,75	123,20	50,45	335,77	52,63
40	15,25	45,92	164,26	51,08	458,37	53,31
50	16,15	46,04	205,33	51,56	582,44	53,83
100	18,95	46,39	410,65	53,07	1214,46	55,42
200	21,78	46,69	821,30	54,57	2503,71	56,99
300	23,44	46,85	1231,95	55,45	3807,23	57,90
400	24,61	46,96	1642,60	56,08	5118,01	58,55
500	25,53	47,04	2053,25	56,56	6433,69	59,04
1000	28,37	44,26	4106,50	58,07	13049,55	60,58
2000	31,22	47,47	8213,00	59,57	26361,82	62,10
3000	32,88	47,58	12319,50	60,45	39719,52	63,00

Table 2: Distance and distance modulus vs cosmic shift.

There have been many other analytic formula proposed to describe the $\mathbb{Z}/\mu$ relation Vigoureux [28], Traunmuller [26], Churoux [7]. It has been shown that their comparison shall be better served by considering the time of flight of the photon t_s instead of the distance modulus. Doing so, Marosi writes that the most important result of the Hubble diagram test is that it leads exactly to the exponential function

$$\mathbb{Z}+1 = e^{2,03x10^{-18} t_s} \tag{5.23}$$

At the speed of light c, whatever the method used to evaluate it, the distance travelled during the time t_s is

$$d = c\ t_s \tag{5.24}$$

$$d = \frac{c}{2,03\ 10^{-18}}\ ln\ (\mathbb{Z}_d + 1) \tag{5.25}$$

$$d = 1,47681\ 10^{26}\ ln\ (\mathbb{Z}_d + 1) \tag{5.26}$$

$$d = 4,786\ ln\ (\mathbb{Z}_d + 1)\ Gpc \tag{5.27}$$

This equation is exactly what our model predicts (2.37) for the local environment ($k = 0$) where the Hubble constant has the value of

$$H_o = 2,03\ x\ 10^{-18}\ sec^{-1} \tag{5.28}$$

$$H_o = 62,6\ Km\ sec^{-1}\ Mpc^{-1} \tag{5.29}$$

It is clear that Marosi analysis confirms the validity of our model being understood that the Hubble constant has the value recently measured by Riess [22]. Marosi [16] concluded by

> *The Hubble diagram test leads to the significant conclusion that either: (1) the universe expanded exponentially during the whole time of its expansion history (at least in the range of $\mathbb{Z} = 0,0141 - 8,1$); or (2) the universe is static and the RS of spectral lines is caused by some as-yet unidentified mechanism.*

This unknown corresponds exactly to our cosmological model.

5.3 Using magnitude

D efining properties of cosmological objects is based on time and space stable and constant values and magnitude is such a property. Magnitude differences between various frequency or wavelength bands such as ultraviolet, blue, visible, infrared etc makes them independent of distance. Objects of the same category shows similar spectrum and are classified on the basis of bandwidth magnitudes. Measuring magnitude with specific frequency bandwidth is nearly equivalent to measuring the mean specific flux at the central frequency of the used filter. But as it has been shown, the specific flux vary with the distance equation (2.63). Finally any property of a cosmological object based on magnitude must take into account the transformation of frequency or wavelength with distance. Not doing so may lead to erroneous conclusions.

5.4 The world we can see

O ur model shows photon transformation along distance, ending when the photon energy correspond to the CMB radiation. At this point photons have a wavelength of $1,873\ x\ 10^7\ \mathring{A}$ corresponding to a temperature of $2,72548\ ^\circ K$. Considering the Hydrogen line $H_\alpha = \lambda_o = 6565\ \mathring{A}$, photons at end of course will have a cosmic-shift of

$$\mathbb{Z}_{cmb} = \frac{\lambda_{cmb} - \lambda_o}{\lambda_o} = \frac{1,873\ x\ 10^7 - 6565}{6565} = 2853 \tag{5.30}$$

Equation (2.30) shows the transformation distance as a function of the cosmic shift

$$d = \eta\ ln\ \{[(\mathbb{Z}_{cmb} + 1)\ +\ \eta\ k]\ /\ [(\mathbb{Z}_{cmb} + 1)/(\mathbb{Z}_d + 1)\ +\ \eta\ k]\} \tag{5.31}$$

whose value at d_{cmb} is

$$d_{cmb} = 32,66\ Gpc = 106\ Gal \tag{5.32}$$

The CMB radiation represents the true limit of the knowledgeable universe, the maximum dimension of the observable universe not its physical dimension. This distance vary upon the wavelength of the photons. It is around 106 Giga light years if we consider the H_α hydrogen line and 224 Giga light years if we consider gamma rays. The following table shows some values highly different from the usual classic value of $13,7$ Giga light years which is nearly fifteen times smaller than the knowledgeable universe.

Line	$\lambda_o\ [\mathring{A}]$	$\mathbb{Z}_{cmb}$	$d_{cmb}\ [Gpc]$	$d_{cmb}\ [Gly]$
γ	1	$1,873\ x\ 10^7$	$68,76$	224
L_∞	912	20 536	$40,77$	133
L_α	1 216	15 402	$39,59$	129
H_∞	3 646	5 136	$35,08$	114
H_α	6 563	2 853	$32,66$	106

Table 3: Maximum observation distances

L et us look at Quasars which are of great luminosity and are usually very far away objects. A value of $Z = 3,638$ has been measured for Quasar Q0201+113 that put it at a relative distance of

$$\frac{d}{D} = \frac{ln(1 + 3,638)}{ln(1 + 2853)} = 0,1929 \tag{5.33}$$

It is about $1/5$ the theoretical observable limit or $6,3\ Gpc$ ($20,5\ Gly$). ULAS J1120+0641 shows a $Z = 7,1$ and is relatively situated at 26% that is $8,59\ Gpc$ or $28\ Gly$

6 Pioneer

N umerous studies about the Pioneer satellites anomaly have been published and this anomaly became an enigma after the incapacity to find a rational explication. The two most recent documents making a complete review and analysis of the numerous proposals are those of Anderson [18] and Turyshev [27]. Clearly detected by 1987, announced at a 1993 Conference Proceedings, Nieto [19] and since the first reference to its presence in a 1994 scientific publication, Nieto [12], it initiated numerous proposals and publications. Most of them concluded to an inertial effect, that is the presence of elements not taken into account by the satellite navigation softwares, translated as a force causing an acceleration of the satellite. The most frequent element suggested as a cause is of thermal nature. This is an error since the power available on the satellite decreases with time while the anomaly stays constant. It is astonishing that most studies always make reference to the Pioneer anomaly as a physical acceleration of the satellite instead of referring to the observed fact of a constant drift of the Doppler signal.

R ecently, thinking differently, Allan Joel Anderson [1] looks for an unknown influence on the electromagnetic signal, that is on the communication link with the satellite. This effect, he calls "Cosmic redshift", is based on the hypothesis of an expanding universe according to the FLRW (Friedman, Lemaître, Robertson, Walker) model. He considers that the Hubble constant "H_o" represents the rate of change of the wavelength of the photons by unit of time. This explication cannot hold because an expanding universe always increases the wavelength or the redshift contrarily to what is observed, a blue shift. But this proposal has the value of pointing attention to a cause acting on the electromagnetic signal itself.

T he presence of very small cyclic variations of the drifting Doppler radio signal have been pointed out, Turyshev [27]. The analysis showed half day, daily, half annual and annual periodicity where the day is the sidereal one, Levy [9]. Those cyclic variations become smaller as the distance to the satellite increase. Isn't this a clear indication of a distance dependency on the light ray length between the satellite and the listening earth stations ?

6.1 Models

T here are two cosmological models supporting an interaction with an electromagnetic signal able to change its wavelength and able to explain the drifting of the Doppler signal. There is the expansionist model also known as the Big Bang and the transformation model. According to both models, the observed wavelength vary as a function of the travelled distance by the electromagnetic wave or, equivalently, as a function of the redshift.

6.1.1 The expansionist model

L et us consider the geometrical space, as isotropic and expanding. Any distance "d", between any fixed and non moving point, remains proportional during expansion.

This is described by a time function "$a(t)$" which acts as a multiplier on all dimensions. However, even if those dimensions change with time, isotropic space implies that relative values of the rate of change will be the same everywhere that is

$$\dot{a}(t)/a(t) = constant \tag{6.1}$$

Considering the distance "d", the wavelength "λ", and the frequency "f", their relative rate of change are

$$\dot{a}/a = \dot{d}/d = \dot{\lambda}/\lambda = -\dot{f}/f \tag{6.2}$$

In this universe we consider that Galaxies don't have intrinsic speed, they move due to the expansion of the universe. Hubble's law links their speed "v" to their distance "d" by the Hubble constant "H_o" as

$$v = H_o d \tag{6.3}$$

In such world, Hubble constant can be related to expansion and to wavelength as

$$H_o = v/d = \dot{d}/d = \dot{\lambda}/\lambda \tag{6.4}$$

Using the redshift definition we get

$$\mathbb{Z} = (\lambda - \lambda_o)/\lambda_o \tag{6.5}$$

$$\lambda = \lambda_o(\mathbb{Z}+1) \tag{6.6}$$

$$\dot{\lambda} = \lambda_o\dot{\mathbb{Z}} \tag{6.7}$$

$$\dot{\lambda}/\lambda = \dot{\mathbb{Z}}/(\mathbb{Z}+1) \tag{6.8}$$

$$H_o = \dot{\lambda}/\lambda = -\dot{f}/f = \dot{\mathbb{Z}}/(\mathbb{Z}+1) \tag{6.9}$$

Wavelength emitted by galactic sources are at a later time always longer. Consequently frequencies are always smaller.

6.1.2 The transformation model

I n our model, we consider that photons transforms naturally without any interaction with other elements of the universe, this being an intrinsic property. Then on their journey, the photon energy lowers while their number increase. This transformation lasts until their wavelength reach the cosmic microwave background (CMB) where it stops. This transformation works inversely for photons whose wavelength is longer than the CMB one. In this model, the Hubble constant comes out naturally where distance has a logarithmic form instead of the classic linear $d = c/H_o \cdot \mathbb{Z}$. According to equation (2.37), it is written for the local universe as

$$d = \pm\, c/H_o \cdot ln(\mathbb{Z}+1) \tag{6.10}$$

The cosmic shift is negative and greater than -1 when wavelengths are longer than the CMB wavelength and positive on the contrary. The variables time "t", distance "d", Hubble constant "H_o", vacuum speed of light "c", cosmic shift "$\mathbb{Z}$", wavelength "λ" and frequency "f" make

$$\mathbb{Z} = exp(\pm H_o d/c) - 1 \tag{6.11}$$
$$\dot{\mathbb{Z}} = \pm H_o(\mathbb{Z}+1) \tag{6.12}$$
$$H_o = \pm \dot{\mathbb{Z}}/(\mathbb{Z}+1) \tag{6.13}$$
$$\lambda = \lambda_o exp(\pm H_o d/c) \tag{6.14}$$
$$\dot{\lambda} = \pm \lambda H_o \tag{6.15}$$
$$f = f_o exp(\pm (-H_o d/c)) \tag{6.16}$$
$$\dot{f} = \pm (-f H_o) \tag{6.17}$$
$$H_o = \pm \dot{\lambda}/\lambda = \pm (-\dot{f}/f) = \pm \dot{\mathbb{Z}}/(\mathbb{Z}+1) \tag{6.18}$$

6.1.3 Comparison

T hese two models set on the same equations except for the negative sign. The expansionist model predicts a constant increase of the wavelength or a frequency decrease. The transformation model shows two solutions depending on the length of the wavelength compared to length of the cosmic microwave background. Larger, there is a decrease and shorter, there is an increase as for the expansionist model. The negative sign and the negative value of the cosmic shift make the difference.

6.2 The Doppler effect

L et us consider a source at rest sending a wave of frequency "f_s" toward an observer also at rest who measure it as an observed frequency "$f_o = f_s$". If that source moves toward this observer, always at rest, with a constant speed "v_s", this observer would measure a different frequency because of the Doppler effect. "c" being the signal speed in the media, the observed frequency is

$$f_o = f_s \cdot c/(c - v_s) \tag{6.19}$$
$$f_s/f_o = 1 - v_s/c \tag{6.20}$$
$$(f_s - f_o)/f_o = -v_s/c \tag{6.21}$$
$$\Delta f/f_o = v_s/c \tag{6.22}$$

If a source at rest is to quickly accelerate during a short time interval "Δt", it will reach the speed "v_s" with an acceleration "$a_s = v_s/\Delta t$". According to Doppler's law a signal emitted by this source now moving at constant speed will be observed at the frequency "f_o". Then we have

$$\Delta f/f_o = a_s \Delta t/c \tag{6.23}$$
$$a_s/c = (\Delta f/\Delta t)/f_o \tag{6.24}$$
$$a_s/c = \dot{f}/f_o \tag{6.25}$$

Without any consideration for the sign, the second term of last equation is the Hubble constant as per the expansionist model or the transformation model. The Pioneer satellite acceleration is then

$$\boxed{a_s = cH_o} \tag{6.26}$$

There stands the product of two natural constants of the universe. Consequently the acceleration attributed to the satellite is an invariant. The Pioneer satellite is clearly in the local universe and using the local Hubble constant $H_o = 2,731933 \cdot 10^{-18}\ s^{-1}$, we find for this universal acceleration of any satellite

$$\boxed{a_s = 8,19 \cdot 10^{-10}\ m\ s^{-2}} \tag{6.27}$$

which is close to the claimed Pioneer anomalous acceleration $8,74 \pm 1,33 \cdot 10^{-10}\ m\ s^{-2}$. This explains the strange numerical coincidence between the speed of light, the Hubble constant and the Pioneer anomaly. There isn't any magic there, no mystery. This is an erroneous association of a non existing inertial force with a cosmological phenomena, a substitution based on the fact that two different interpretations, an inertial and a cosmological one, both leading to the same Doppler effect.

6.3 Analysis

A s we have shown, both models, expansion or transformation, lead to the same mathematical expression for the Hubble constant, exception being made of the sign. Both models predict the same cosmic redshift as long as the wavelength of the signal is longer than the CMB one. For the expansion model, there is no limit to this evolution of the wavelength. For the transformation model, this evolution stops at the CMB wavelength. And conversely, longer wavelengths evolve toward this CMB wavelength. And it is exactly the situation in which the satellites evolve. The frequency of the signal used between the satellite and the earth stations is lower than the CMB one. This is why with increasing distance the drift of the Doppler signal is toward the blue (shorter wavelength or higher frequency). It is impossible for the expansion model to cope with this fact because it always predicts an increase of the wavelength. Since the satellite is moving at a constant speed "v_P", we write "$d = v_P\ t$" into equation (6.11) and take its derivative against time

$$\mathbb{Z} = exp(-\ H_o\ v_P\ t/c) - 1 \tag{6.28}$$

$$\dot{\mathbb{Z}} = -\ (\mathbb{Z}+1)\ (H_o/c)\ v_P \tag{6.29}$$

Then it is clear that the Doppler drift is proportional to the satellite distance from the observer and to the satellite speed. It shall be noted that "$\mathbb{Z}$" is a negative quantity that becomes more negative as the satellite goes farther from the observer.

I t is important to keep in mind that the fundamental point is the line of sight distance or the optical path between the earth and the satellite. The important change of the Doppler drift when a satellite encounters a planet (flyby) is explained by the abrupt change of the satellite direction and speed causing a new rate of change of the optical path or the line of sight distance to the satellite.

7 Conclusion

T he expansionist model of cosmology also called the "Big Bang" is always a speculative one. Instead of compounding with an elastic relativistic metric with adjustable parameters, we find more plausible our model based exclusively on Maxwell electromagnetism and the quantum world. Contrarily to tired light models it doesn't blur images but redden them.

O ur model shows that cosmological distances can be measured according to a logarithmic law. It gives a sound basis to two Hubble constants related to a dissipation constant. With a null value, we observe a local Hubble constant of $84,3\ km\ sec^{-1}\ Mpc^{-1}$. Riess [22] measured the Hubble constant as $73,02\ km\ sec^{-1}\ Mpc^{-1}$ for which we associate the value of $k = \frac{5}{3}\ x\ 10^{-29}\ m^{-1}$ for the dissipation constant.

W e compounded the Hubble constant and its local version as a function of three other fundamental constants. The computed values are exactly the same as the measured ones.

W e defined the local Hubble length whose value is $3,5563$ Giga parsecs. We also defined the local Hubble surface ${}^{k0}\sigma_H$ whose value is exactly $2\pi\ 10^{19}\ m^2$. This surface is equivalent to that of a sphere of radius $\sqrt{5}\ Gm$ or $0,015$ AU.

W e reviewed some problematic cases for the expansionist model and shown that they are naturally explained by our model.

W e showed that Marosi [16] statistical analysis of cosmological candles follows the same logarithmic law as predicted by our model so confirming the wandering of expansion cosmologies.

W e computed the maximal dimension of the observable universe. It is not limited to $13,7$ billion light years but knowledgeable up to hundred billion light years.

W e showed that the Pioneer satellite anomaly is the substitution of a non existent acceleration for a transformation of the communication signal. This Pioneer pseudo acceleration is a universal constant and the same for any satellite.

Y ou may visit our web site at http://www.phrenocarpe.org

References

[1] A.J. Anderson. The measurement of the hubble constant H_o in the solar system. *https://arxiv.org/abs/1011.1944v1*, nov 2010.

[2] H. Arp. Arguments for a Hubble constant near H_0=55. *The Astrophysical Journal*, 571:615–618, jun 2002. http://iopscience.iop.org/0004-637X/571/2/615/fulltext/15433.text.html.

[3] Jeffrey A. Willick Puneet Batra. A determination of the hubble constant from cepheid distances and a model of the local peculiar velocity field. *http://arxiv.org/abs/astro-ph/0005112*, feb 2001.

[4] R.J. Bouwens and al. Photometric constraints on the redshift of z=10 candidate UDFJ-39546284 from deeper WFC3/IR+ACS+IRAC observations over the HUDF. *http://arxiv.org/abs/1211.3105*, feb 2013.

[5] G.B. Brammer and al. A tentative detection of an emission line at 1,6 μm for the z=12 candidate UDFJ-39546284. *http://arxiv.org/abs/1301.0317*, jan 2013.

[6] E. Cameron and T. Petitit. On the evidence for cosmic variation of the fine structure constant : a Bayesian reanalysis of the quasar dataset. *http://arxiv.org/abs/1207.6223*, jul 2012.

[7] P. Churoux. A new interpretation of the hubble law. *Journal of Modern Physics, 6, 1227-1232, 2015.*, 2015. http://dx.doi.org/10.4236/jmp.2015.69127.

[8] CODATA. Physical constants. *http://physics.nist.gov/constants*, 2010.

[9] A. Levy B. Christophe P. Bério G. Métris J-M. Courty and S. Reynaud. Pioneer 10 doppler data analysis: disentangling periodic and secular anomalies. *https://arxiv.org/pdf/0809.2682v2.pdf*, jan 2009.

[10] D.J. Fixsen. The temperature of the cosmic microwave background. *IOP Science*, 707:916, dec 2009. http://iopscience.iop.org/0004-637X/707/2/916.

[11] E.P. Hubble. A relation between distance and radial velocity among extragalactic. *Proc. Nat. Accad. Sci.*, 15:168–173, 1929.

[12] M.M. Nieto T. Goldman J.D. Anderson E.L. Lau and J. Perez-Mercader. Theoretical motivation for gravitational experiments on ultra-low energy antiprotons and anti hydrogen. *https://arxiv.org/abs/hep-ph/9412234*, mar 1994.

[13] G. Lemaître. The gravitational field in a fluid sphere of uniform invariant density, according to the theory of relativity. *PhD thesis, Massachusetts Institute of Technology*, 1927.

[14] G. Lemaître. Un univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nébuleuses extra-galactiques. *Ann. Soc. Sci. Brux.*, 47:49–59, dec 1927.

[15] G. Lemaître. Expansion of the universe, a homogeneous universe of constant mass and increasing radius accounting for the radial velocity of extra-galactic nebulae. *MNRAS*, 91:483–490, 1931.

[16] L.A. Marosi. Modelling and analysis of the hubble diagram of 280 snia supernovae and gamma ray bursts redshifts with analytical and empirical redshiftmagnitude functions. *International Journal of Astronomy and Astrophysics*, 6:272–275, aug 2016.

[17] NASA. Universe older than previously thought. *NASA Science News*, mar 2013. http://science.nasa.gov/science-news/science-at-nasa/2013/21mar_cmb.

[18] J.D. Anderson P.A. Laing E.L. Lau A.S. Liu M.M. Nieto and S.G. Turyshev. Study of the anomalous acceleration of the Pioneer 10 and 11. *https://arxiv.org/abs/gr-qc/0104064v5*, mar 2005.

[19] M.M. Nieto. The quest to understand the pioneer anomaly. *https://arxiv.org/abs/gr-qc/0702017v1*, fev 2007.

[20] V.N. Yershov V.V. Orlov and A.A. Raikov. Possible signature of distant foreground in the planck data. *http://arxiv.org/pdf/1410.5814v1.pdf*, 10 2014.

[21] A.A. Penzias and R.W. Wilson. A measurement of excess antenna temperature at 4080 mc/s. *Astrophysical Journal*, 142:419–421, 1965. http://adsabs.harvard.edu/abs/1965ApJ...142..419P.

[22] A.G. Riess and al. A determination of the local value of the hubble constant. *http://arxiv.org/pdf/1604.01424v1.pdf*, apr 2016.

[23] S.W. Jha S. Dhawan and B. Leibundgut. Measuring the hubble constant with type ia supernovae as near-infrared standard candles. *http://arxiv.org/pdf/1707.00715v1.pdf*, jul 2017.

[24] I. Steer. Le Maitre's limit. *http://arxiv.org/abs/1212.6566*, dec 2012.

[25] H. Terazawa. Environment dependant fundamental physical constants. *http://arxiv.org/abs/1202.1859*, feb 2012.

[26] H. Traunmuller. From magnitudes and redshifts of supernovae, their light-curves and angular sizes of galaxies to a tenable cosmology. *Astrophysics and Space Science*, 350:755–767, apr 2014.

[27] S.G. Turyshev and V.T. Toth. The pioneer anomaly. *Living Reviews in Relativity*, jan 2010. http://arxiv.org/abs/1001.3686.

[28] J.-M. Vigoureux D. Vigoureux P. Vigoureux and M. Langlois. An analytical expression for the hubble diagram of supernovae and gamma-ray bursts. *https://arxiv.org/pdf/1411.3648v1.pdf*, nov 2014.

[29] M. Kunz W. Cardona and V. Pettorino. Determining h0 with bayesian hyper-parameters. *http://arxiv.org/pdf/1611.06088v2.pdf*, nov 2016.

[30] Wikipedia. E.P. Hubble. *http://en.wikipedia.org/wiki/Edwin_Hubble*, 2000.

[31] Wikipedia. Physical constants. *http://en.wikipedia.org/wiki/physical_constant*, 2000.

A Constants

Constants and values used throughout this document.

Constant	Symbol	Value	Units
Vacuum light speed	c	$2,99792458\ x\ 10^{8}$	m^{-1}
Gravitational	G	$6,67384\ x\ 10^{-11}$	$kg^{-1}\ m^{3}\ s^{-2}$
Planck	h	$6,62606957\ x\ 10^{-34}$	$kg\ m^{2}\ s^{-1}$
Reduced Planck	h	$1,054571726\ x\ 10^{-34}$	$kg\ m^{2}\ s^{-1}$
Fine structure	α	$7,2973525698\ x\ 10^{-3}$	
Reduced fine structure	$\tilde{\alpha}$	$1,161409733\ x\ 10^{-3}$	
Rydberg	R_{∞}	$1,0973731568539\ x\ 10^{7}$	m^{-1}
Reduced Rydberg	$\tilde{R_{\infty}}$	$1,74652362\ x\ 10^{6}$	m^{-1}
Plank length	l_p	$1,616199\ x\ 10^{-35}$	m
Reduced Plank length	$\tilde{l_p}$	$2,57226059\ x\ 10^{-36}$	m
Astronomical Unit	AU	$1,495979\ x\ 10^{11}$	m
Parsec	pc	$3,085678\ x\ 10^{16}$	m
Parsec	pc	$2,062648\ x\ 10^{5}$	AU
Parsec	pc	$3,261507$	ly
Light year	ly	$9,460895\ x\ 10^{15}$	m
Sidereal year	sy	$3,155815\ x\ 10^{7}$	s

Table 4: Constants part I

Constant	Symbol	Value	Units
Balmer	H_α	$6,5646\ x\ 10^{-7}$	m
Lyman	L_α	$1,2157\ x\ 10^{-7}$	m
CMB wavelength	λ_{cmb}	$1,873\ x\ 10^{-3}$	m
CMB frequency	ν_{cmb}	$1,602\ x\ 10^{11}$	Hz
CMB temperature	T_{cmb}	$2,72548$	K
Boltzmann	B	$1,380660\ x\ 10^{-23}$	$J\ K^{-1}$
Dissipation constant	k	$(5/3)\ x\ 10^{-29}$	m^{-1}
Hubble constant	H_o	$2,365923\ x\ 10^{-18}$	s^{-1}
Hubble constant	H_o	$73,0$	$km\ s^{-1}\ Mpc^{-1}$
Local Hubble constant	$^{k0}H_o$	$2,731933\ x\ 10^{-18}$	s^{-1}
Local Hubble constant	$^{k0}H_o$	$84,3$	$km\ s^{-1}\ Mpc^{-1}$
Hubble length	l_H	$1,267127\ x\ 10^{26}$	m
Hubble length	l_H	$4,1065$	Gpc
Local Hubble length	$^{k0}l_H$	$1,097364\ x\ 10^{26}$	m
Local Hubble length	$^{k0}l_H$	$3,5563$	Gpc
Local Hubble surface	$^{k0}\sigma_H$	$2\pi\ x\ 10^{19}$	m^2

Table 5: Constants part II

B Other ways

H ere we give two other ways to consider the photons evolution on very long distances. The first method use Maxwell electromagnetic equations and the second method is based on a numerical sequence of photons mutations. Both methods produce the same conclusions.

B.1 Using electromagnetism

T he vacuum properties of an electromagnetic wave at very far distances are unknown to us. We suppose they are the same as they are locally meaning that Maxwell's laws of electromagnetism are the same everywhere in the universe. Then for a plane wave moving in the direction $\vec{k}$, the electrical field $\vec{E}$ and the magnetic field $\vec{H}$ are dependent on distance " d " and time " t ".

$$\vec{E} = \vec{i}\, E_x(d,t) \tag{B.1}$$

$$E_x = E\ exp\ [j\omega(t - \frac{d}{c}) + \theta] \tag{B.2}$$

$$\vec{H} = \vec{j}\, H_y(d,t) \tag{B.3}$$

$$H_y = H\ exp\ [j\omega(t - \frac{d}{c}) + \theta] \tag{B.4}$$

The Poynting vector represents the energy flux carried by the wave

$$\vec{\mathbb{S}} = \vec{E} \times \vec{H} \tag{B.5}$$

which is for the plane wave

$$\vec{\mathbb{S}} = \frac{E^2}{\mu_o c}\, \vec{k} \tag{B.6}$$

Between extremely distant points the Poynting vector cannot represent the energy conservation principle. A redshift is observed meaning a variation of the wavelength, an absent parameter of $\mathbb{S}$. Meanwhile the Poynting vector certainly represents the mean energy carried by the photons of the wave each being of energy

$$E = \hbar\ \omega \tag{B.7}$$

When considering extremely long distances it might be appropriate to consider the variation of the photon density N and it's energy level so that the energy regime is preserved and the quantity

$$\xi = N\hbar\omega \tag{B.8}$$

is kept constant on long distances while N and ω vary according to the distance d. Bringing together those two quantities

$$\mathbb{S} = \xi \tag{B.9}$$

$$\frac{E^2}{\mu_o c} = \hbar N(d)\omega(d) \tag{B.10}$$

$$E = (\mu_o c\hbar N(d)\omega(d))^{\frac{1}{2}} \tag{B.11}$$

shows an electrical field E varying as a function of the photon density N and the circular frequency ω both being dependent on the distance d. Then the components of the electromagnetic wave are

$$E_x = (\mu_o c\hbar N(d)\omega(d))^{\frac{1}{2}}\ exp[j\omega(d)(t-\frac{d}{c})+\theta] \tag{B.12}$$

$$H_y = (\frac{\hbar N(d)\omega(d)}{\mu_o c})^{\frac{1}{2}}\ exp[j\omega(d)(t-\frac{d}{c})+\theta] \tag{B.13}$$

Simplifying the writing

$$E_x = F_d\ exp\ [j\omega_d(t-\frac{d}{c})+\theta] = F_d exp\ [\cdot] \tag{B.14}$$

$$H_y = G_d\ exp\ [j\omega_d(t-\frac{d}{c})+\theta] = G_d exp\ [\cdot] \tag{B.15}$$

and knowing that

$$\frac{\partial E_x}{\partial d} = -\mu_o\frac{\partial H_y}{\partial t} \tag{B.16}$$

we get

$$\frac{\partial E_x}{\partial d} = \frac{\partial F_d}{\partial d}\ exp\ [\cdot] + jF_d\ exp\ [\cdot]\ \{\frac{\partial\omega_d}{\partial d}(t-\frac{d}{c}) - \frac{\omega_d}{c}\} \tag{B.17}$$

$$\frac{\partial H_y}{\partial t} = G_d\ exp\ [\cdot]\ \{j\omega_d\} \tag{B.18}$$

Since that at any point in space

$$\frac{E}{H} = \frac{F_d}{G_d} = \mu_o c \tag{B.19}$$

$$\therefore \quad G_d = \frac{F_d}{\mu_o c} \tag{B.20}$$

then

$$\frac{\partial F_d}{\partial d} + jF_d\ \frac{\partial\omega_d}{\partial d}\ (t-\frac{d}{c}) = 0 \tag{B.21}$$

Considering

$$E = F = |(\mu_o c\hbar N_d\omega_d)^{\frac{1}{2}}| \tag{B.22}$$

one get

$$\left\{N_d\ \frac{\partial\omega_d}{\partial d} + \omega_d\ \frac{\partial N_d}{\partial d}\right\} + j\ \left\{2N_d\omega_d\ \frac{\partial\omega_d}{\partial d}\ (t-\frac{d}{c})\right\} = 0 \tag{B.23}$$

The real part between braces may be obtained differently by considering the fact that the quantity ξ (B.8) doesn't vary with distance so it's derivative is null and we get

$$\frac{\partial\xi}{\partial d} = N_d\ \frac{\partial\omega_d}{\partial d} + \omega_d\ \frac{\partial N_d}{\partial d} = 0 \tag{B.24}$$

From here we follow the same path as presented in the main part of this document from equation (2.10) where $k=0$ and get the known results.

B.2 Numerically

S pectral properties of atoms are well known in the laboratory. But when we observe them from far distances they show a redshift of their wavelength. Individual photons are characterized by a wavelength λ_o and energy E_o

$$E_o = \frac{hc}{\lambda_o} \tag{B.25}$$

Let us consider a cohort of N_o photons per unit volume showing an energy per unit of volume G_o

$$G_o = N_o E_o \tag{B.26}$$

After a time T there are $N_k > N_o$ photons per unit volume showing an energy per unit of volume G_k

$$G_k = N_k E_k \tag{B.27}$$

According to the principle of conservation of energy we write

$$N_k E_k = N_o E_o \tag{B.28}$$

Here we suppose that the transformation of the photons happens by successive leaps. A first one produces a new photon and the energy in the group is re-balanced. This process is the same for all other transformations happening in the group. After k transformations the energy of $N_o + k$ photons becomes

$$E_k = \frac{N_o E_o}{N_o + k} \tag{B.29}$$

After k transformations the number of new photons is

$$k = N_o \left\{ \frac{E_o}{E_k} - 1 \right\} \tag{B.30}$$

$$k = N_o \left\{ \frac{\frac{hc}{\lambda_o}}{\frac{hc}{\lambda_k}} - 1 \right\} \tag{B.31}$$

$$k = N_o \, \frac{\lambda_k - \lambda_o}{\lambda_o} \tag{B.32}$$

$$k = N_o \, \mathbb{Z}_k \tag{B.33}$$

where we made use of the redshift $\mathbb{Z}_k$. Then the photon density is

$$N_k = N_o + k \tag{B.34}$$

$$N_k = N_o + N_o \, \mathbb{Z}_k \tag{B.35}$$

$$N_k = N_o \, (\mathbb{Z}_k + 1) \tag{B.36}$$

The spectral line intensity I_k is proportional to the number of photons per unit of volume and it also increases the same way as

$$I_k = I_o\ (\mathbb{Z}_k + 1) \tag{B.37}$$

and the photon energy is

$$E_k = \frac{E_o}{\mathbb{Z}_k + 1} \tag{B.38}$$

and the wavelength is

$$\lambda_k = \lambda_o\ (\mathbb{Z}_k + 1) \tag{B.39}$$

B etween successive transformations the cohort moves a distance Δd. Without saying anything about the way such transformations happen, we suppose that the tension pushing the photons to transform is proportional to the actual photon density N_k. The number of new photons Δk per unit of distance is

$$\frac{\Delta k}{\Delta d} \propto N_k \tag{B.40}$$

and inversely, the distance of transformation is given by

$$\frac{\Delta d}{\Delta k} \propto \frac{1}{N_k} \tag{B.41}$$

which says that the distances where the transformations happens are inversely proportional to the photon density which constantly increases upon distance. If b is the proportionality constant and $N_k = N_o + k$ we have for small intervals

$$\frac{\partial d}{\partial k} = \frac{b}{N_o + k} \tag{B.42}$$

Upon integration

$$d = b\ ln(N_o + k) + Cte \tag{B.43}$$

The initial conditions being $d = 0$ and $k = 0$ then

$$Cte = -b\ ln\ N_o \tag{B.44}$$

and the distance is

$$d = b\ ln\ \frac{N_o + k}{N_o} \tag{B.45}$$

Using equation (B.33) we write

$$d = b\ ln\ (\mathbb{Z} + 1) \tag{B.46}$$

from which the redshift and the wavelength as a function of distance are

$$\mathbb{Z} = e^{\frac{d}{b}} - 1 \tag{B.47}$$

$$\lambda_d = \lambda_o \, e^{\frac{d}{b}} \tag{B.48}$$

Expanding the exponential as a series and keeping the first order terms for small distances

$$\mathbb{Z} = \frac{d}{b} \tag{B.49}$$

There we recognize Hubble's law and we set

$$b = \frac{c}{H_o} \tag{B.50}$$

From this point we compute the same equations as in the preceding section that is to say equation (2.37).